Molecular Bose-Einstein condensate as an amplifier of weak interactions

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Abstract
Collisions and chemical reactions of molecules in Bose-Einstein condensates (BECs) are extremely sensitive to weak fields. This sensitivity arises due to the high density of compound resonances and a macroscopic number of molecules with kinetic energy $E = 0$ (perfect energy resolution). We suggest that chemical reactions in molecular BECs could be used to enhance effects produced by small external perturbations and search for a parity-violating energy difference in chiral molecules.

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Techniques to trap and cool atoms and molecules are rapidly developing. Bose-Einstein condensation (BEC) has been realized in numerous dilute atomic gases [1]. While a pure molecular BEC has not yet been achieved, ultra-cold molecules have been formed [2] and, just recently, a coherent superposition of an atomic and a molecular BEC was reported [3].

Collisions of atoms in a BEC can be controlled by the variation of a relatively weak magnetic field; the magnitude and sign of the scattering length can be changed by varying the field near a Feshbach resonance [4, 5]. In this way the expansion and collapse of an atomic BEC (termed "Bosenova") has been observed [6]. In collisions of molecules in a BEC, a change of the scattering length could be achieved with a magnetic field much weaker than that used to obtain the same effect in atoms. This is because there is an exponential dependence of the density of resonances on the number of "active" particles. This high sensitivity to weak fields could be used to search for a parity violating energy difference between chiral molecules.

It is well-known that biological molecules have a definite chiral structure (for example, there are only naturally occurring left-handed amino acids and right-handed sugars) [7]. There have been numerous attempts to explain this effect by the influence of the parity violating weak interaction, which breaks the energy equivalence of right- and left-handed molecules [8, 9]. That parity violation can discriminate between molecules of different chirality is easily seen: a parity violating nuclear spin-independent electron-nucleus interaction in atoms [10, 11] creates a spin helix of the electrons which interacts differently with right- and left-handed molecules. However, the parity violating energy difference ΔE_{PV} is very small [12],

$$\Delta E_{PV} \sim 10^{-20} Z^5 \eta \text{ a.u.} , \qquad (1)$$

where Z is the nuclear charge of the heaviest atom, and η is an asymmetry factor which can be found from molecular structure calculations. This strong dependence on Z originates from the weak ($\propto Z^3$) and spin-orbit ($\propto Z^2$) interactions. It may appear that in molecules with heavy atoms ΔE_{PV} could become relatively large due to the Z^5-dependence. However, the geometrical suppresion factor η remains very small. We should note that the effect may be larger for molecules with two heavy atoms [13]. For recent calculations of ΔE_{PV} for various molecules, see, e.g., [14] and references therein.

Let us consider how a parity violating energy difference could manifest itself in the collision of two molecules in a BEC. Remember that in order to form a chiral molecule there must be at least four atoms involved [15]; therefore the collision of two diatomic molecules is sufficient. The distance between energy levels in a combined molecule D is much smaller than the distance in an initial molecule, $D \ll D_i$. Therefore, in an "ordinary" system it would be impossible to resolve compound resonances of the combined system since we would expect $k_B T \gg D$, where k_B is the Boltzmann constant, T is the temperature. However, in a molecular BEC a finite fraction of molecules (up to 100% for $T \ll T_c$) have zero kinetic energy, $E = 0$ [16]. This means that we have a possibility of perfect energy resolution [17]. This allows one to study molecular collisions and chemical reactions in a unique situation where the reaction is dominated by the closest compound resonance of the combined system.

We can express the cross-section for formation of a chiral compound molecular state by the Breit-Wigner

formula

$$\sigma = \frac{\pi}{k^2} \frac{\Gamma_c \Gamma}{(E - E_0)^2 + \Gamma^2/4} \ , \qquad (2)$$

where k is the wave vector, Γ_c is the capture width, and Γ is the total width of the resonance. The parity violating weak interaction in the chiral molecules shifts the resonance energies, for example, let's consider that for the left-handed structure $E_0 \to E_L = E_0 - \Delta E_{PV}/2$ while for the right-handed structure $E_0 \to E_R = E_0 + \Delta E_{PV}/2$. Therefore, cross-sections for the formation of left and right molecules, σ_L and σ_R, from achiral components may be different. We can define an asymmetry parameter

$$P = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R} \ . \qquad (3)$$

The maximum value for P is reached when $E = E_0 \pm \Gamma/2$. At this energy the asymmetry parameter

$$|P_{\max}| = \frac{\Delta E_{PV}}{\Gamma} \ . \qquad (4)$$

The resonances can be shifted close to zero energy (the energy of the molecular collisions) $E_0 \pm \Gamma/2 = 0$ by application of an external electric or magnetic field.

The width of the level Γ may be quite small since the capture width $\Gamma_c = 0$ for energy $E = 0$. The radiative width may be smaller than 10^7Hz. This can be compared to the weak energy shift. The largest values for ΔE_{PV} that have been considered in molecular calculations are $\sim 10^4$Hz (e.g., for H_2Po_2 [14]).

Another possibility to observe parity violation in molecules may be related to the admixture of an s-wave to a p-wave compound resonance. For energy $E = 0$, only s-wave molecules have a significant interaction cross section. Consider now a p-wave compound resonance. It seems to be invisible for $E = 0$. However, the weak interaction W mixes states of opposite parity and produces the combined state $|\psi\rangle = |p\rangle + \beta|s\rangle$, thus opening the s-wave reaction amplitude proportional to β. The mixing coefficient $\beta = \langle p|W|s\rangle/(E_s - E_p)$ is enhanced since the energy interval between the opposite parity compound states $(E_s - E_p)$ is very small due to the high level density in a combined molecule. Note that this mechanism is responsible for the enhancement of weak interaction effects in neutron-nucleus reactions $\sim 10^6$ times [18] (for a review of the experiments, see [19]). Interference of the very small p-wave amplitude and the weak-induced s-wave amplitude may possibly also lead to a difference in the production of right and left molecules in the p-wave resonance.

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[15] Note that it is not necessary to have all four atoms different. Even such simple molecules as H_2O_2 have a chiral structure.

[16] Strictly speaking, the kinetic energy is not exactly zero due to the finite size of the system. The ground state of the molecules, however, is very close to $E = 0$ and the kinetic energy spread is vanishingly small ($\Delta E \sim \hbar^2/MR^2$, where M is the molecular mass, R is the size of the system).

[17] Colliding molecules can be in an internal excited state. However, if $D_i > k_B T$ the probability of the excited state is small. Moreover, the molecules which are not in the ground state will probably not be trapped and will quickly leave the system.

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